Causal Inference What If - Chapter 6

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Overview

1. DAG examples (pp. 69∼75)
2. D-separation (pp. 76∼77)
3. Systematic bias (pp. 78∼82)

Section 1

DAG examples (pp. 69∼75)

What is DAG

$$A \longrightarrow Y$$

Figure 6.2

- A is temporally prior to Y
- There is a **direct causal effect** for at least one individual.
- Square box : restriction (=condition)

DAG's examples

Between **A** & **Y**...

- B : Mediator

$$A \longrightarrow B \longrightarrow Y$$

Figure (자작)

A and **Y** : Causal effect O, Associated O

$$A \longrightarrow \boxed{B} \longrightarrow Y$$

Figure 6.5

A and **Y** : Causal effect O, Associated ×

DAG's examples

Between **A** & **Y**...

- L : Common Cause

$$\begin{tikzcd} L \arrow[r] \arrow[rr, bend left] & A & Y \end{tikzcd}$$

Figure 6.3

A and **Y** : Causal effect $\times$, Associated O

$$\begin{tikzcd} \boxed{L} \arrow[r] \arrow[rr, bend left] & A & Y \end{tikzcd}$$

Figure 6.6

A and **Y** : Causal effect $\times$, Associated $\times$

DAG's examples

Between **A** & **Y**...

- L : Colider

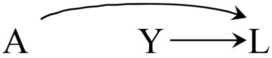

Figure 6.4

A and **Y** : Causal effect $\times$, Associated $\times$

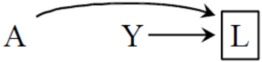

Figure 6.7

A and **Y** : Causal effect $\times$, Associated O

DAG's examples

Between **A** & **Y**...

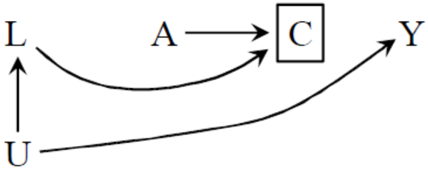

Figure 8.3

Are A and Y associated?

Section 2

D-separation (pp. 76∼77)

D-separation

Process to define D-separation

1. Define **“path”** with length n
2. Define **“blocked”** on length 2 path
3. Define **“blocked”** on arbitrary path
4. Define **“D-separation”** on DAG

D-separation

1. Define **"path"** with length n

- $V_0 \sim V_1 \sim \cdots \sim V_n, \quad n \geq 1,$ No duplicate

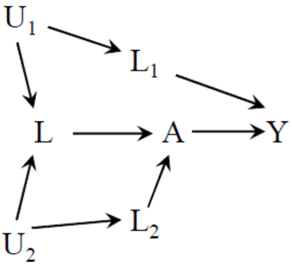

Figure 7.6

- Example 1 : $L \sim A \sim Y$
- Example 2 : $L_2 \sim U_2 \sim L \sim U_1 \sim L_1 \sim Y$

D-separation

2. Define **"blocked"** on length 2 path

	Mediator	Common Cause	Colider
w/o conditioned	Opened $A \rightarrow B \rightarrow Y$ Figure (자작)	Opened $\begin{tikzcd} L \arrow[r] \arrow[rr, bend left] & A & Y \end{tikzcd}$ Figure 6.3	Blocked $\begin{tikzcd} A \arrow[rr, bend left] & Y \arrow[r] & L \end{tikzcd}$ Figure 6.4
w/ conditioned	Blocked $A \rightarrow \boxed{B} \rightarrow Y$ Figure 6.5	Blocked $\begin{tikzcd} \boxed{L} \arrow[r] \arrow[rr, bend left] & A & Y \end{tikzcd}$ Figure 6.6	Opened $\begin{tikzcd} A \arrow[rr, bend left] & Y \arrow[r] & \boxed{L} \end{tikzcd}$ Figure 6.7

D-separation

3. Define **“D-separation”** on DAG

- length 1 path : Opened

- length 2 path : already defined

- length n path $(n > 2)$:
  - Path has $n - 1$ length 2 sub-paths
  - Path is blocked **iff** some length 2 length 2 sub-paths is blocked

4. Define **“D-separation”** on DAG

- A is **d-separated** from Y conditional of L

  iff **Every paths** between A and Y are **blocked** conditional of L

D-separation

Pearl (1988) proved the following fundamental theorem :

A is d-separated from Y conditional of L

$\Longrightarrow$ **A and Y are independent(=not associated) given L**

Converse holds under **"Faithfulness conditions".**

Discovery of causal structure

Discovery : Data $\implies$ DAG

(1) Find **conditional independence** relationships

(2) Find **d-separated** relationships

(3) Make DAG with **time** information and **d-separated** relationships

- (1) $\implies$ (2) : Faithfulness conditions
- (2) $\implies$ (3) : Often impossible because it's not identifiable

Section 3

Systematic bias (pp. 78∼82)

Systematic bias

We say there is **systematic bias** when data are insufficient to identify the causal effect even with an infinite sample size.

- (Unconditional) Bias :

$$
\begin{aligned}
&\Pr\left[Y^{a=1}=1\right]-\Pr\left[Y^{a=0}=1\right] \\
&\neq \Pr[Y=1 \mid A=1]-\Pr[Y=1 \mid A=0]
\end{aligned}
$$

- Conditional Bias :

$$
\begin{aligned}
&\Pr\left[Y^{a=1}=1 \mid L=l\right]-\Pr\left[Y^{a=0}=1 \mid L=l\right] \\
&\neq \Pr[Y=1 \mid L=l, A=1]-\Pr[Y=1 \mid L=l, A=0]
\end{aligned}
$$

Systematic bias

There are Three types of systematic bias

- Confounding : Chapter 7
- Selection Bias : Chapter 8
- Measurement Bias : Chapter 9